

# Comparative analysis of calculation methods for overlapping area in antenna placement problem using brute force algorithm

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**Abstract**— This paper presents a method to reduce the computational complexity of calculation the criterion function in a brute force algorithm to solve the antenna placement problem in a grid model. Calculating overlap zones between antennas is one of the criteria for optimizing wireless network topology. We present a comparative analysis of two methods for the calculating of this criterion. The impact of software implementation of these methods on computational complexity is discussed and theoretical estimates are confirmed by experimental results. The modified method involves performing an initial calculation of overlaps between antennas in all possible positions and storing this data in a multidimensional array, which allows for quick access during the main algorithm process. Our research focuses on a modified method that can reduce the computational burden of repetitive computations during the search for an optimal solution. The experimental results confirm that the presented modified method effectively reduces computing times in calculating the criteria function. This allows us to expand the possibilities of using the enumeration algorithm to solve large-scale optimization problems.

**Keywords**— *combinatorial optimization problem, brute force algorithm, antennas placement problem, computational complexity, overlapping area.*

## I. INTRODUCTION

The Antenna placement problem (APP) is a critical challenge in wireless network design, aiming to maximize coverage while minimizing overlap and interference. Given the rising demands of customers, mobile operators are compelled to deploy multiple carriers and sectors, significantly expanding the number of cells. This expansion results in base stations overlapping with each other. The large number of cells makes manual solutions impractical, requiring advanced computing systems and optimized algorithms. Mobile network operators are increasingly shifting towards automated planning to minimize computation time, enhance network performance [1].

Combinatorial optimization problem (COP) is a branch of optimization in applied mathematics. COPs include choosing the best combination of elements or allocation of resources, entities, or components from a finite set to optimize a given objective, such as maximizing profit or minimizing cost in various problems such as Quadratic Assignment Problem [2], Unbounded Knapsack Problem and so on. Whether it's

positioning sensors in a network, or arranging components on a circuit board, the goal is to find the most efficient arrangement that minimizes costs, maximizes coverage, or optimizes some other performance metric [3].

The problem of wireless network topology optimization can be modeled as a COP. When multiple antennas are allocated within a confined area, calculating their overlapping regions for all possible placements and determining the best position of antennas among all possible antenna placements to achieve criteria such as minimizing overlaps and maximizing coverage [4].

Calculating overlapping areas between geometric shapes, such as circles or polygons, is widely used in various fields, including computer graphics, geographical information systems (GIS), and wireless communications [5]. Various algorithms have been developed to compute these overlaps, considering factors like complexity, accuracy, and adaptability to different shapes and configurations.

Intersection calculations involve geometrical problems, such as determining overlapping areas of multiple circles (or other shapes), which becomes computationally intensive when the number of antennas and mesh size increase. The search space grows as factorial of the number of antennas and possible placements positions, making APP an NP-hard problem. The increasing in complexity for calculation of criteria functions makes it difficult to find optimal solutions in a reasonable computation time, which requires the development of more efficient methods.

Heuristic methods [6] offer an alternative approach by efficiently exploring the solution space and providing approximate solutions efficiently within a reasonable amount of time but do not guarantee optimality. Among the various exact methods, brute force algorithm (BFA) guarantee to find the exact solution but may be computationally expensive for large tasks due to the combinatorial nature of the problems.

The problem of solving the APP problem using BFA is determined by the need to calculate the overlapping areas for all possible antenna positions, which becomes computationally complex for large grid sizes. For each pair of antennas, the overlap area must be recalculated many times for all possible antenna positions, which leads to redundant computations and high processing costs.

This paper introduces a modified method to reduce the computational complexity of the criterion calculation when

using the brute force algorithm in solving APP, which is based on the initial calculation the antennas overlap without compromising the exhaustive nature of the search. The experimental results of the comparison of the modified and the original methods are presented as well as an analysis of the differences in their effectiveness.

## II. RELATED WORKS

Interest in APP has grown significantly over the past few years, due to its importance in various communication areas. But the corresponding planning problem is computationally NP-hard and involves analyzing of all possible variants of antenna placement options. Various innovative methods and algorithms have been presented in different works for reducing computational complexity and finding optimal or exact solution when solving various APP and COPs.

The paper [7] presents an iterative algorithm to efficiently compute the intersection areas of multiple circles. This problem is significant in scenarios such as wireless communications, where overlapping coverage areas of antennas need to be analyzed to evaluate network performance. The proposed algorithm uses a trellis structure, which simplifies the calculation by breaking down the problem into smaller subsets of intersecting circles. Using two geometric theorems to reduce computational resources allows handling configurations involving a large number of circles. The authors demonstrate the effectiveness of the algorithm and address the challenges of ensuring accuracy while minimizing computational cost, presenting a detailed analysis of the algorithm's complexity through various wireless network applications.

The paper [8] develops and analyzes optimization methods for antenna topologies to enhance radar imaging quality and detection performance, introducing a topology optimization technique for optimal arrangement of transmitters and receivers in MIMO radar systems. Various optimization strategies, including the BFA, Linear and Non-linear Least Squares (LLS and NLLS), Genetic Algorithms (GA), and Direct Search (DS) methods, are explored for their effectiveness. The brute force approach systematically evaluates all possible combinations of antenna positions, ensuring global optimum but becoming computationally intensive with more antennas. To address this issue, LLS and NLLS leverage mathematical constraints to reduce the search space and speed up convergence.

Authors [9] present some optimization methods for reducing computational complexity when solving the QAP and similar COPs, which do not lead to loss of accuracy using a brute force algorithm. In this article, the traditional approach based on parallelization of resource-intensive computations, and methods to accelerate the process of generating variants and limiting their number have been investigated and the results of computational experiments confirm the effectiveness of the proposed approach and its applicability in sequential and parallel implementations of algorithm.

This paper [10] presents the application of combinatorial optimization algorithms in the context of positioning radio

stations for wireless and mobile networks and focuses on addressing the challenges of optimizing the placement of base stations (BTS) in mobile telephony to enhance network coverage and efficiency. By integrating various optimization methods, the study aims to improve the positioning of base stations in a region, optimizing signal coverage and network performance.

The paper [11] focuses on using GA to solve the problem of optimal antenna placement for maximizing wireless coverage. By employing a GA, the authors aim to address the non-linear and complex nature of antenna optimization, which is challenging using traditional methods. The authors design fitness functions that evaluate placement configurations based on coverage, signal strength, and overlap minimization. The results demonstrate that the GA can find superior configurations compared to heuristic methods, especially when dealing with high-density environments and multiple antennas.

This paper [12] studies the application of branch and bound algorithms to solve the APP in Cellular networks. Based on a branch and bound search technique, the authors use combinatorial optimization and graph theory to model the problem, enabling a structured approach to handle complex placement scenarios. The study compares different heuristic and exact optimization methods to find the best antenna configurations. The paper's results demonstrate significant improvements in coverage and network performance compared to traditional placement strategies.

The study [13] highlights the strengths and weaknesses of each algorithm, providing practical guidance for selecting suitable techniques based on problem size and complexity. This paper presents the effectiveness of several stochastic algorithms - Genetic Algorithm, Evolutionary Strategy, Simulated Annealing, and Hill Climbing in optimizing the placement of multiple antennas on a single platform. The objective is to find the best antenna configurations that minimize mutual coupling and maximize the radiation pattern for each antenna.

The paper [14] presents a comprehensive overview of optimization algorithms specifically designed to enhance the lifetime of wireless sensor networks (WSNs) and discusses the strengths and weaknesses of various approaches, highlighting trade-offs in terms of complexity, computational efficiency, and scalability.

The paper [15] focuses on the critical aspect of determining the overlapping coverage areas in cellular networks, which is essential for optimizing performance and minimizing interference. The authors employ geometric principles to derive equations that accurately represent the coverage and interference patterns and present analytical models that enable the calculation of overlapping regions between adjacent cells, which is particularly important in dense network deployments.

The paper [16] presents a robust solution to perform set operations on polygons, which are fundamental in fields such as computer graphics and geographic information systems (GIS). The authors introduce a scan-line algorithm designed

for efficient computation of the intersection, union, and difference between two polygons, achieving a time complexity of  $O((n+m+k) \log(n+m+k))$  in the worst case. The efficiency of the proposed algorithm is a significant improvement over existing methods, primarily because it consolidates multiple operations into a single process, thereby minimizing computational redundancy.

Geometric algorithms, such as the scan-line algorithm [17], have been proposed to enhance the efficiency of overlap calculations. This method processes shapes in sorted order, reducing the number of comparisons required by maintaining active intervals of overlapping edges. By sweeping across the plane and updating active edges, the scan-line algorithm can efficiently compute intersections and overlaps, significantly reducing computational overhead.

In this paper [18] various optimization techniques are applied to antenna design and placement problems. It covers classic methods such as gradient-based approaches, GA, and simulated annealing, along with emerging strategies like machine learning and hybrid methods. The authors emphasize how these techniques address challenges such as minimizing interference, optimizing radiation patterns, and enhancing coverage. The paper also highlights the use of advanced numerical tools, including electromagnetic solvers, to support optimization processes.

The paper [19] presents brute force algorithms as a basis for comparing different access points (AP) placement strategies. The authors recognize the limitations of brute force and present alternative strategies that balance computational efficiency with solution quality. For small-scale AP placements, brute force can serve as a useful tool for benchmarking, but for many AP, heuristic and approximation algorithms are recommended. In this context, the paper introduces more efficient methods such as the Shadowing Elimination Search and the Check Line-of-Sight Status algorithms, which aim to reduce the search space significantly by focusing only on configurations that are likely to improve line-of-sight coverage.

In our study we use overlap region calculations at the initial stage of the algorithm and store these values in a three-dimensional matrix structure. Instead of recalculating them on each iteration, the approach simply retrieves these values from the lookup table. This method allows to significantly reducing the computation complexity of calculation the criterion during BFA search. The algorithm still explores all possible antenna placement options, preserving the exhaustive nature of the BF approach, but does so with greatly reduced computational overhead for each evaluation.

### III. CALCULATION OF OVERLAP AREAS

Figure 1 shows the problem of calculating the distance and overlap area between two omnidirectional antennas.

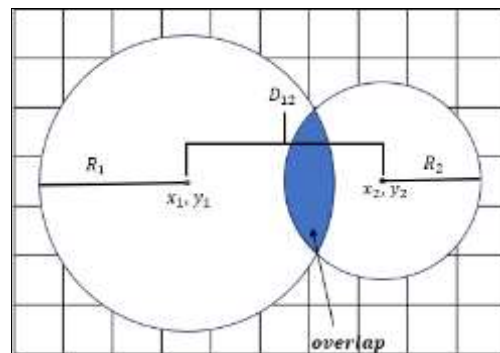


Fig. 1. Overlap area

The mathematical solution to this problem when solving the APP is formulated as follows:

- We assume that the reliable signal transmission between antennas is ensured when the following condition is met:

$$R \leq R_{max} \quad (1)$$

- Distance between antennas centers in Euclidean metric:

$$D_{12} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (2)$$

If  $D_{12} \geq (R_1 + R_2)$ , the distance between two antennas positions is greater than or equal to the total range of these antennas, so the antennas do not overlap, and the overlap area is zero:

$$overlap = 0 \quad (3)$$

In case the large antenna ( $R_1$ ) completely covers the small one ( $R_2$ ), i.e.:  $D_{12} \leq (R_1 - R_2)$ , then:

$$overlap = \pi \cdot R_2^2 \quad (4)$$

Otherwise, then  $D_{12} < (R_1 + R_2)$ , the overlap area can be calculated using the geometric approach:

$$\alpha_1 = 2 \cdot \arccos\left(\frac{R_1^2 - R_2^2 + D_{12}^2}{2 \cdot R_1 \cdot D_{12}}\right)$$

$$\alpha_2 = 2 \cdot \arccos\left(\frac{R_2^2 - R_1^2 + D_{12}^2}{2 \cdot R_2 \cdot D_{12}}\right)$$

$$overlap = 0.5 \cdot R_1^2(\alpha_1 - \sin(\alpha_1)) + 0.5 \cdot R_2^2(\alpha_2 - \sin(\alpha_2)) \quad (5)$$

The computational complexity of this method seriously affects the performance of BFA. We propose a modified approach that involves calculating distances and overlap areas between antennas before running the BFA. The obtained results are stored in a three-dimensional matrix, where the first index represents the position of the first antenna and the second index represents the position of the second antenna. The third index defines overlapping antennas. For this we use the unique number of the overlapping pair. For example, for four antennas ( $N = 4$ ) - A, B, C, D - six combinations are possible: AB, AC, AD, BC, BD and CD, numbered from 1 to 6 respectively.

As the number of antennas ( $N$ ) increases, the number of possible combinations ( $K$ ) increases accordingly:

$$K(N) = (N \cdot (N - 1)) / 2 \quad (6)$$

Formula (6) counts unique pairs without repetitions, ensuring that each combination is considered only once.

This matrix is used as a lookup table to quickly obtain overlap values without recalculating them on each BFA iteration. For example, if antenna A is placed at position (1,1) and B is placed at position (3,3), the matrix can immediately provide the corresponding overlap area without calculation using formulas (1) – (5). This approach reduces the computational complexity the calculation of criterion function and simplifies the implementation of BFA.

#### IV. EXPERIMENTAL TASK

In our experiments, we use omnidirectional antennas with the following characteristics:

TABLE I. CHARACTERISTICS OF OMNIDIRECTIONAL ANTENNAS

Antenna types	Coverage range (km) - (R)
A	4
B	3
C	2
D	1

Antennas of all types are located in the center of the grid model cell. The total number of cells (M) for an 8x8 square grid is 64, and for 10x10 grid it is 100. For numerical experiments, we use the following parameters, as shown in Table II.

TABLE II. TERMS OF THE PROBLEM

Task 1		Task 2		Task 3	
Type	M	Type	M	Type	M
A	64	A	100	A	100
B		B		B	
C		C		C (two)	
D		D		D	

Figure 2 shows the overlap areas for random antenna placement (Task 3), where antenna type A is green, B is red, C antennas are yellow and D is purple.

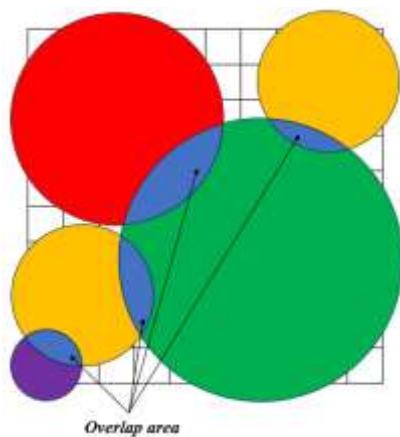


Fig. 2. Random placement of antennas for Task 3.

#### V. BRUTE FORCE ALGORITHM

For each placement option, the algorithm calculates overlap areas between the antennas using a geometric approach. The BFA approach becomes computationally

intensive as the number of antennas and grid size increase, with a time complexity of  $O(M!/(M - N)!)$ .

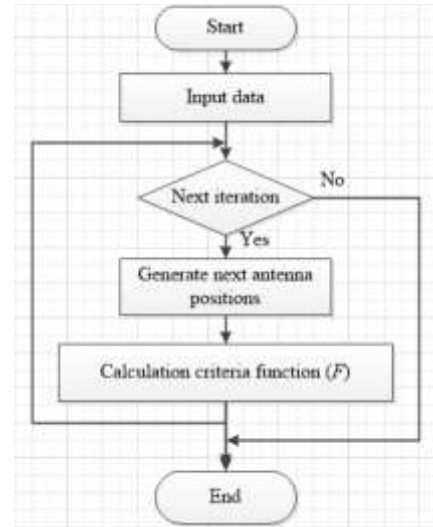


Fig. 3. Original BFA for APP solution.

Figure 3 shows the flow chart of BFA for solving APP with overlap area as the criterion.

#### VI. SPEEDING UP THE CALCULATION OF OVERLAPPING AREAS

The choice of performance improvement methods depends on the specific characteristics of problem, its size and complexity, and the required criterion functions. To improve the original method, we propose to use criterion calculation before running BFA. Initial calculation and storage the overlapping area play a crucial role in reducing the computational complexity of calculation criterion function in APP. Pre-computing the distances and overlapping areas between each pair of antenna positions allows us to do this only once and store the results in a lookup table.

##### A. Initial calculation and memorization of distance matrix

The initial calculation serves as a fundamental step in the modified method. This avoids re-calculating the distance between (equation 2) antennas when calculating the criterion function during BFA. This not only reduces the computational complexity, but also speeds up the process, providing the required distance faster.

##### B. Initial calculation and memorization of overlap matrix

This process involves calculating and storing overlapping areas between all possible antenna locations and can significantly reduce the calculation of overlapping areas (equations 3-5) and can bypass redundant calculations.

We implemented initial calculation as:

- The APP parameters are defined as: M is the number of cells, N is the number of antennas, R is the vector of antenna ranges,  $D[M][M]$  is the matrix of distance between antennas, K – the number of antenna combinations,  $OV[M][M][K]$  is the matrix of overlap areas between antennas, and P is the antenna placement vector.

- Analyze all antenna positions on the grid map.
- Calculate the distance and overlap between the antennas and store these values in the distance matrix  $D$  and overlap matrix  $OV$ .

We believe that this approach significantly reducing redundant calculations and speeding up the brute force process.

## VII. IMPLEMENTATION OF BRUTE FORCE ALGORITHM FOR APP SOLUTION

BFA guarantees an optimal solution but can be quite inefficient for large tasks due to their exhaustive search nature.

TABLE III. MOIFICATION OF ORIGINAL METHOD

Step	Original	Modified
0		<b>Initial calculation</b> of matrix $D$ and $OV$
1	Generate next vector $P$	Generate next vector $P$
2	<b>Calculate</b> overlapping areas	<b>Read</b> overlapping areas value
3	Break 1 if not END	Break 1 if not END
4	Output result	Output result

We present an approach to reduce computations and reuse intermediate results to decrease the complexity of computing the criterion function. Table III shows the comparison between the original and modified methods.

## VIII. EXPERIMENTAL RESULTS

We tested the effectiveness of the modified version in practice by implementing BFA in the C++ programming language in the Visual Studio environment, using 64-bit release mode with maximum optimizations disabled.

The results are shown in Table IV.

TABLE IV. EXPERIMENTAL RESULTS

Task (N, M, K)	Calculation time (sec)		Speedup
	Original	Modified	
Task 1 (4, 64, 6)	7.08	0.44	16.09
Task 2 (4, 100, 6)	34.02	2.74	12.42
Task 3 (5, 100, 10)	5473.62	432.83	12.65

The presented results clearly confirm our proposal. The modified method reduces the computational complexity of calculating the criteria function, providing an acceleration of more than 12 times.

## IX. CONCLUSION

This study demonstrates the significant performance improvement achieved by implementing an initial calculation of distances and overlapping areas. Of course, the effectiveness of this approach depends on the complexity of calculating the criterion function and the number of placement options. For the APP the complexity of calculating the criterion function is determined by equations (3) – (5), and the number of placement options is equal to  $M!/(M - N)!$ .

Removing the calculation of overlapping areas from the inner circle of BFA when solving APP can improve the efficiency of wireless network design and management. This

approach is especially valuable for dynamic configurations with varying numbers of antennas and grid sizes.

Our future research will focus on extending proposed approach to parallel computing technologies, which will allow handling larger grid sizes and significantly increasing the number of antennas considered. The use of parallelism will further reduce the computation time and improve the performance for large-scale antenna placement problems, opening the way for solving many practical COPs.

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